

What is claimed is:

1. A method for designing apparatus for use in a magnetic resonance system for receiving a magnetic resonance signal having a predetermined radio frequency, said apparatus and said magnetic resonance system having a common longitudinal axis, said method comprising designing the apparatus by treating the apparatus as a transmitter of a radio frequency field having the predetermined radio frequency and then designing said transmitter by:

(a) defining a target region in which the radial magnetic component of the radio frequency field is to have desired values, said target region surrounding said longitudinal axis;

(b) specifying desired values for said radial magnetic component of the radio frequency field at a preselected set of points within the target region;

(c) determining a complex current density function  $\mathbf{J}$ , having real and imaginary parts, on a specified cylindrical surface by:

(i) defining the complex current density function as a sum of a series of basis functions multiplied by complex amplitude coefficients having real and imaginary parts; and

(ii) determining values for the complex amplitude coefficients using an iterative minimization technique applied to a residue vector obtained by taking the difference between calculated field values obtained using the complex amplitude coefficients at the preselected points and the desired values at those points; and

(d) converting said complex current density function  $\mathbf{J}$  into a set of capacitive elements located on the specified cylindrical surface and a set of inductive elements located on the specified cylindrical surface by:

(i) converting the complex current density function into a curl-free component  $\mathbf{J}_{\text{curl-free}}$  and a divergence-free component  $\mathbf{J}_{\text{div-free}}$  using the relationships:

$$\mathbf{J}_{\text{curl-free}} = \nabla \psi, \text{ and}$$

$$\mathbf{J}_{\text{div-free}} = \nabla \times \mathbf{S},$$

where  $\psi$  and  $\mathbf{S}$  are functions obtained from the complex current density function through the equations:

$$\nabla^2 \psi = \nabla \cdot \mathbf{J},$$

$$-\nabla^2 \mathbf{S} = \nabla \times \mathbf{J}, \text{ and}$$

$$-\nabla^2(\mathbf{n} \cdot \mathbf{S}) = \mathbf{n} \cdot \nabla \times \mathbf{J},$$

where  $\mathbf{n}$  is a vector normal to the specified cylindrical surface;

- (ii) calculating locations on the specified cylindrical surface for the set of capacitive elements by contouring the function  $\psi$ ; and
- (iii) calculating locations on the specified cylindrical surface for the set of inductive elements by contouring the function  $\mathbf{n} \cdot \mathbf{S}$ .

2. The method of Claim 1 wherein the basis functions are triangular and pulse functions.
3. The method of Claim 1 wherein the iterative minimization technique is selected from the group consisting of linear steepest descent and conjugate gradient descent.
4. The method of Claim 1 wherein (i) the set of inductive elements on the specified cylindrical surface defined first and second ends for the apparatus and (ii) the target region has a midpoint that is closer to the first end than to the second end.
5. The method of Claim 1 comprising the additional step of displaying the locations on the specified cylindrical surface for the set of inductive elements.
6. The method of Claim 1 comprising the additional step of producing the set of inductive elements and the set of capacitive elements on the specified cylindrical surface.
7. A method for designing apparatus for use in a magnetic resonance system for transmitting a radio frequency field or both transmitting a radio frequency field and receiving a magnetic resonance signal, said apparatus and said magnetic resonance system having a common longitudinal axis, said method comprising:
  - (a) defining a target region in which the radial magnetic component of the radio frequency field is to have desired values, said target region surrounding said longitudinal axis;
  - (b) specifying desired values for said radial magnetic component of the radio frequency field at a preselected set of points within the target region;
  - (c) defining a target surface external to the apparatus on which the magnetic component of the radio frequency field is to have a desired value of zero at a preselected set of points on said target surface;
  - (d) determining a first complex current density function, having real and imaginary parts, on a first specified cylindrical surface and a second complex current

density, having real and imaginary parts, on a second specified cylindrical surface, the radius of the second specified cylindrical surface being greater than the radius of the first specified cylindrical surface by:

(i) defining each of the complex current density functions as a sum of a series of basis functions multiplied by complex amplitude coefficients having real and imaginary parts; and

(ii) determining values for the complex amplitude coefficients using an iterative minimization technique applied to a first residue vector obtained by taking the difference between calculated field values obtained using the complex amplitude coefficients at the set of preselected points in the target region and the desired values at those points and a second residue vector equal to calculated field values obtained using the complex amplitude coefficients at the preselected set of points on the target surface; and

(e) converting said first and second complex current density functions into sets of capacitive elements and sets of inductive elements located on the specified cylindrical surfaces by:

(i) converting each of the first and second complex current density functions into a curl-free component  $\mathbf{J}_{\text{curl-free}}$  and a divergence-free component  $\mathbf{J}_{\text{div-free}}$  using the relationships:

$$\mathbf{J}_{\text{curl-free}} = \nabla \psi, \text{ and}$$

$$\mathbf{J}_{\text{div-free}} = \nabla \times \mathbf{S},$$

where  $\psi$  and  $\mathbf{S}$  are functions obtained from the respective first and second complex current density functions through the equations:

$$\nabla^2 \psi = \nabla \cdot \mathbf{J},$$

$$-\nabla^2 \mathbf{S} = \nabla \times \mathbf{J}, \text{ and}$$

$$-\nabla^2 (\mathbf{n} \cdot \mathbf{S}) = \mathbf{n} \cdot \nabla \times \mathbf{J},$$

where  $\mathbf{n}$  is a vector normal to the respective first and second specified cylindrical surfaces and  $\mathbf{J}$  is the respective first and second complex current density functions;

(ii) calculating locations on the respective first and second cylindrical surfaces for the respective sets of capacitive elements by contouring the respective functions  $\psi$ ; and

(iii) calculating locations on the respective first and second cylindrical surfaces for the respective sets of inductive elements by contouring the respective functions  $\mathbf{n} \cdot \mathbf{S}$ .

8. The method of Claim 7 wherein the basis functions are triangular and pulse functions.

9. The method of Claim 7 wherein the iterative minimization technique is selected from the group consisting of linear steepest descent and conjugate gradient descent.

10. The method of Claim 7 wherein (i) the set of inductive elements on the first specified cylindrical surface define first and second ends for the apparatus and (ii) the target region has a midpoint that is closer to the first end than to the second end.

11. The method of Claim 7 comprising the additional step of displaying the locations of the sets of inductive elements on the first and second specified cylindrical surfaces.

12. The method of Claim 7 comprising the additional step of producing the sets of inductive and capacitive elements on the first and second specified cylindrical surfaces.

13. A method for designing apparatus for use in a magnetic resonance system for receiving a magnetic resonance signal having a predetermined radio frequency, said apparatus and said magnetic resonance system having a common longitudinal axis, said method comprising designing the apparatus by treating the apparatus as a transmitter of a radio frequency field having the predetermined radio frequency and then designing said transmitter by:

(a) defining a target region in which the radial magnetic component of the radio frequency field is to have desired values, said target region surrounding said longitudinal axis;

(b) specifying desired values for said radial magnetic component of the radio frequency field at a preselected set of points within the target region;

(c) determining a complex current density function  $\mathbf{J}$ , having real and imaginary parts, on a specified cylindrical surface by:

(i) defining the complex current density function as a sum of a series of basis functions multiplied by complex amplitude coefficients having real and imaginary parts; and

(ii) determining values for the complex amplitude coefficients by solving a matrix equation of the form:

$$[\mathbf{A}](\mathbf{a}^C) = \mathbf{B}$$

where  $\mathbf{A}$  is a transformation matrix between current density space and magnetic field space whose components are based on time harmonic Green's functions,  $\mathbf{a}^C$  is a vector of the

unknown complex amplitude coefficients, and  $\mathbf{B}$  is a vector of the desired values for the magnetic field specified in step (b), said equation being solved by:

- (1) transforming the equation into a functional that can be solved using a preselected regularization technique, and
- (2) solving the functional using said regularization technique to obtain values for the complex amplitude coefficients; and
- (d) converting said complex current density function into a set of capacitive elements located on the specified cylindrical surface and a set of inductive elements located on the specified cylindrical surface.

14. The method of Claim 13 where the regularization functional is chosen so as to minimize the integral of the dot product of the complex current density function with itself over the specified cylindrical surface.

15. The method of Claim 13 where the complex amplitude coefficients are chosen so that the complex current density function has zero divergence.

16. The method of Claim 13 wherein step (d) is performed by:

- (i) converting the complex current density function into a curl-free component  $\mathbf{J}_{\text{curl-free}}$  and a divergence-free component  $\mathbf{J}_{\text{div-free}}$  using the relationships:

$$\mathbf{J}_{\text{curl-free}} = \nabla \psi, \text{ and}$$

$$\mathbf{J}_{\text{div-free}} = \nabla \times \mathbf{S},$$

where  $\psi$  and  $\mathbf{S}$  are functions obtained from the complex current density function through the equations:

$$\nabla^2 \psi = \nabla \cdot \mathbf{J},$$

$$-\nabla^2 \mathbf{S} = \nabla \times \mathbf{J}, \text{ and}$$

$$-\nabla^2 (\mathbf{n} \cdot \mathbf{S}) = \mathbf{n} \cdot \nabla \times \mathbf{J},$$

where  $\mathbf{n}$  is a vector normal to the specified cylindrical surface;

- (ii) calculating locations on the cylindrical surface for the set of capacitive elements by contouring the function  $\psi$ ; and
- (iii) calculating locations on the cylindrical surface for the set of inductive elements by contouring the function  $\mathbf{n} \cdot \mathbf{S}$ .

17. The method of Claim 13 wherein the radio frequency field has a wavelength  $\lambda$ , the inductive elements on the cylindrical surface define a longitudinal length L, and

$$L \geq 0.2 \lambda.$$

18. The method of Claim 13 where the predetermined radio frequency is at least 80 megahertz.

19. The method of Claim 13 wherein (i) the set of inductive elements on the specified cylindrical surface define first and second ends for the apparatus and (ii) the target region has a midpoint that is closer to the first end than to the second end.

20. The method of Claim 13 comprising the additional step of displaying the locations for the set of inductive elements on the specified cylindrical surface.

21. The method of Claim 13 comprising the additional step of producing the set of inductive elements and the set of capacitive elements on the specified cylindrical surface.

22. A method for designing apparatus for use in a magnetic resonance system for transmitting a radio frequency field or both transmitting a radio frequency field and receiving a magnetic resonance signal, said apparatus and said magnetic resonance system having a common longitudinal axis, said method comprising:

(a) defining a target region in which the radial magnetic component of the radio frequency field is to have desired values, said target region surrounding said longitudinal axis;

(b) specifying desired values for said radial magnetic component of the radio frequency field at a preselected set of points within the target region;

(c) defining a target surface external to the apparatus on which the magnetic component of the radio frequency field is to have a desired value of zero;

(d) determining a first complex current density function, having real and imaginary parts, on a first specified cylindrical surface and a second complex current density, having real and imaginary parts, on a second specified cylindrical surface, the radius of the second specified cylindrical surface being greater than the radius of the first specified cylindrical surface by:

(i) defining each of the complex current density functions as a sum of a series of basis functions multiplied by complex amplitude coefficients having real and imaginary parts; and

(ii) determining values for the complex amplitude coefficients by simultaneously solving matrix equations of the form:

$$\left[ A_I^C \right] \left( a^C \right) + \left[ A_I^S \right] \left( a^S \right) = B^C$$

$$\left[ A_2^C \right] \left( a^C \right) + \left[ A_2^S \right] \left( a^S \right) = B^S$$

where  $A_I^C$ ,  $A_I^S$ ,  $A_2^C$ , and  $A_2^S$  are transformation matrices between current density space and magnetic field space whose components are based on time harmonic Green's functions,  $a^C$  and  $a^S$  are vectors of the unknown complex amplitude coefficients for the first and second complex current density functions, respectively,  $B^C$  is a vector of the desired values for the radial magnetic field specified in step (b), and  $B^S$  is a vector whose values are zero, said equations being solved by:

(1) transforming the equations into functionals that can be solved using a preselected regularization technique, and

(2) solving the functionals using said regularization technique to obtain values for the complex amplitude coefficients; and

(e) converting said first and second complex current density functions into sets of capacitive elements and sets of inductive elements located on the specified cylindrical surfaces.

23. The method of Claim 22 where the regularization functional is chosen to as to minimize the integral of the dot product of the first complex current density function with itself over the first specified cylindrical surface and to minimize the integral of the dot product of the second complex current density function with itself over the second specified cylindrical surface.

24. The method of Claim 22 where the complex amplitude coefficients are chosen so that the first and second complex current density functions each has zero divergence.

25. The method of Claim 22 wherein step (e) is performed by:

(i) converting each of the first and second complex current density functions into a curl-free component  $\mathbf{J}_{\text{curl-free}}$  and a divergence-free component  $\mathbf{J}_{\text{div-free}}$  using the relationships:

$$\mathbf{J}_{\text{curl-free}} = \nabla \psi, \text{ and}$$

$$\mathbf{J}_{\text{div-free}} = \nabla \times \mathbf{S},$$

where  $\psi$  and  $\mathbf{S}$  are functions obtained from the respective first and second complex current density functions through the equations:

$$\nabla^2 \psi = \nabla \cdot \mathbf{J},$$

$$-\nabla^2 \mathbf{S} = \nabla \times \mathbf{J}, \text{ and}$$

$$-\nabla^2 (\mathbf{n} \cdot \mathbf{S}) = \mathbf{n} \cdot \nabla \times \mathbf{J},$$

where  $\mathbf{n}$  is a vector normal to the respective first and second specified cylindrical surfaces and  $\mathbf{J}$  is the respective first and second complex current density functions;

(ii) calculating locations on the respective first and second cylindrical surfaces for the respective sets of capacitive elements by contouring the respective functions  $\psi$ ; and

(iii) calculating locations on the respective first and second cylindrical surfaces for the respective sets of inductive elements by contouring the respective functions  $\mathbf{n} \cdot \mathbf{S}$ .

26. The method of Claim 22 wherein the radio frequency field has a wavelength  $\lambda$ , the inductive elements on the first cylindrical surface define a longitudinal length  $L_1$ , the inductive elements on the second cylindrical surface define a longitudinal length  $L_2$ , and

$$L_1 \geq 0.2 \lambda, \text{ and}$$

$$L_2 \geq 0.2 \lambda.$$

27. The method of Claim 22 where the frequency of the radio frequency field is at least 80 megahertz.

28. The method of Claim 22 wherein (i) the set of inductive elements on the first specified cylindrical surface define first and second ends for the apparatus and (ii) the target region has a midpoint that is closer to the first end than to the second end.

29. The method of Claim 22 comprising the additional step of displaying the locations of the sets of inductive elements on the first and second specified cylindrical surfaces.



30. The method of Claim 22 comprising the additional step of producing the sets of inductive and capacitive elements on the first and second specified cylindrical surfaces.

31. A method of converting a complex current density function  $\mathbf{J}$  into sets of capacitive and inductive elements located on a specified cylindrical surface comprising:

(i) converting the complex current density function into a curl-free component  $\mathbf{J}_{\text{curl-free}}$  and a divergence-free component  $\mathbf{J}_{\text{div-free}}$  using the relationships:

$$\mathbf{J}_{\text{curl-free}} = \nabla \psi, \text{ and}$$

$$\mathbf{J}_{\text{div-free}} = \nabla \times \mathbf{S},$$

where  $\psi$  and  $\mathbf{S}$  are functions obtained from the complex current density function through the equations:

$$\nabla^2 \psi = \nabla \cdot \mathbf{J},$$

$$-\nabla^2 \mathbf{S} = \nabla \times \mathbf{J}, \text{ and}$$

$$-\nabla^2 (\mathbf{n} \cdot \mathbf{S}) = \mathbf{n} \cdot \nabla \times \mathbf{J},$$

where  $\mathbf{n}$  is a vector normal to the specified cylindrical surface;

(ii) calculating locations on the cylindrical surface for the set of capacitive elements by contouring the function  $\psi$ ; and

(iii) calculating locations on the cylindrical surface for the set of inductive elements by contouring the function  $\mathbf{n} \cdot \mathbf{S}$ .

32. The method of Claim 31 comprising the additional step of displaying the locations of the set of inductive elements on the specified cylindrical surface.

33. The method of Claim 31 comprising the additional step of producing the set of inductive elements and the set of capacitive elements on the specified cylindrical surface.